Topological properties of high-voltage electrical transmission networks

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Abstract. The topological properties of high-voltage electrical power transmission networks in several UE countries (the italian 380 kV, the french 400 kV and the spanish 400 kV networks) have been studied from available data. A static estimate of the vulnerability of the networks has been given by measuring the level of damage introduced by a controlled removal of links. Results suggest the hypothesis that constraining the growth mechanisms of complex networks might result in weakening their structures. Topological studies could be useful to make vulnerability assessment and to design specific action to reduce structural weaknesses.

Keywords: Information theory, combinatorial optimisation, graphs and networks

1. Introduction

Recent electrical power outages experienced in the north-east area of US and in Canada (August 14, 2003) and on a large portion of the italian country (September 28, 2003) have strongly pushed forward the problem of the analysis, the control and the fast healing of complex critical infrastructures, after the occurrence of some type of fault. The analysis issued after these events [1,2] have highlighted some key points deserving a careful consideration:

(1) complex networks, such as power transmission lines, are often scarcely fault-tolerant; their topology (compelled by the specific distribution of sources and loads in a country) might be prone to weaknesses from the topological up to the electrical levels;

(2) there is a lack of instruments for the on-line analysis of networks functionality which may rapidly provide decision support for healing (or self-healing) actions; complexity, in large networks, originates non-linear and non-local effects which must be treated with suitable approaches which consider the network at large;

(3) there is a very high level of interconnection between critical infrastructures whose consequences cannot be easily predicted on the basis of simple models; tight inter-dependencies between networks are at the origin of *cascading failures* (the failure of one induces malfunctions into the others which, in turn, feed back, with further events, the network where failures initiated etc.). This is a further origin of *network's vulnerability* which should attract the necessary attention from both the theoretical and the technological sides.

Although the solution of the problems issued by these points is far to be simple and achieved, several novel theoretical advancements might help in sheding a new light on this matter [3].

It has been noticed that many "real-world" networks arising from *non-supervised* growth processes display common topological features [4-6]. Networks in biology (metabolic and protein-protein interaction networks), in sociology (scientific publication co-authors network), in communication engineering (the Internet), in the information society (the web pages), although describing different systems and phenomena, show similarities in their structure [5]. The most striking among them is the emergence of a network topology with a distribution of the node's degree k, P(k), characterized by a power-law function of the type [4-6]

$$P(k) \sim k^{-\gamma} \tag{1}$$

with the value of γ restricted in a narrow interval ($2<\gamma<3$) [5]. The structure of the networks originating power-law distribution of node's degree is "scale-free" [3,4], where highly (*hubs*) and loosely (*leaves*) connected nodes coexist. These exhibit a number of interesting functional properties which have been widely discussed [5].

Although diverse types of networks display a scale-free topology, many others cannot be structured that way, due to some geometrical constraint [7]. It has been noticed, in fact, that "pure" 2-dimensional networks (such as roads network, the underground maps, electrical lines, railways networks etc.) have a *single-scale* behavior (i.e. their degree distribution is an exponential) which, moreover, is truncated at some value of the degree, as arbitrarily large values of their degrees cannot be attained for structural reasons [7] (for instance, higher than fivefold road junctions are unlike to occur etc.). Self-organized networks with a scale-free topology, due to their relevance for allowing the understanding of the origin of their "common" organization and the presence of an underlying growth mechanism, have attracted, in recent years, much more interest than exponential or random networks. However, the advancements made in the topological and the spectral analysis of these systems, can be easily transferred to other specifics, to helping the description and the

analysis of objects such as the electrical power networks with a single-scale structure. These, as those previously referred to as purely 2-dimensional, have a more regular structure, characterized by low values of the node's degree and showing a cumulative degree distribution with exponential or gaussian-type decay [3,7].
In this work, we analyze the structure and the related properties of the italian, the french and the spanish high-voltage electrical power transmission lines (the italian 380 kV, the french and the spanish 400 kV networks). Data have been taken from a coarse-grain description, under the form of network's graphs, displayed in the web sites of the national agencies operating the electric transmission lines in the different countries [8]. Maps have been transformed into graphs from which adjacency matrices have been deduced. In the italian case, the availability of official documents containing the description of the whole electrical transmission network (comprising lines from 380 kV down to 120 kV) allowed to make a deeper assessment of the properties of a more realistic network, not limited to a single voltage. The analysis of a single voltage line, however, should not be regarded as a weakness of the present study. It has to be reminded that the highest voltage networks, although being composed by a small number of nodes and links, are the most important components of the transmission networks which are totally ineffective in absence of the correct functioning of the highest voltage component.

Several properties of the networks have been evaluated from these data: beside the description of the general topological features (average degree, degree's distribution and its cumulative distribution function), we have performed a detailed analysis of properties such as the distribution of the node's distance and, by a suitable application of the spectral analysis, of properties related to network’s vulnerability. At the end, we will attempt to sketch a number of actions to be taken to reduce the impact of network's weaknesses on their vulnerability. These actions might be viewed as a possible way in which the "complex systems view" of the electrical power networks might be useful for helping the design of a new generation of tools for the analysis of complex systems and for supporting the (human or cyber) operator in the control of critical infrastructures.

2. Definition and evaluation of network's properties

Data have been taken from ref.[8], where they are stored under the form of pictures representing the national maps, displaying the location of the nodes and links of their high-voltage networks. In these maps, nodes are power sources (thermal and hydroelectric plants) and substations, links represent transmission lines of 2 types: single- and double-circuit lines. Raw data have been suitably transformed into a mathematical description of the networks, each identified by a set of data, G_1, G_2 and G_3 (the I-380 kV, F-400 kV and S-400 kV networks, respectively) where $G_i=(N_i,L_i)$ (N_i and L_i are the number of nodes and links). The fine-grain italian network (comprising lines from 380 kV down to 120 kV) will be referred to as G_4. The fine-grain network G_4 has been analyzed as if it represent the highest-voltage network G_1 at a later stage of growth, in order to assess the variation of the principal network’s properties. The sizes of these networks are reported in the first two columns of Table1.

One of the mathematical objects allowing a complete definition of the network is the *Adjacency Matrix* **A**; if the network has undirected and unitary links, assumed to hold hereafter, it is defined as $A_{ij}=1$ if nodes *i* and *j* are connected, 0 otherwise. All the relevant properties of the network can be deduced from the analysis of the Adjacency matrix [1,9]. A further matrix which can be associated to the network, which will be used in this work, is the so-called *Laplacian Matrix* **L**, defined as **L** = **D** - **A** (where **D** is the diagonal matrix having $D_{ii}=k_i$, with k_i defined as the degree of the i-th node). Further insights on the structure and the properties of the networks can be gained by the spectrum analysis of the **A** and/or the **L** matrices [5,10].

Topological quantities are determined to gain a first insight on the network's static properties. We have evaluated the clustering coefficient *c* [4,11] (the average value of the fraction of links present between the first neighbors of a node with respect to the maximum possible number of links between them); the network diameter *d,* defined as the maximum point-to-point distance in the network; the characteristic path length $<l>$, which is the averaged value of the point-to-point distance over the $N(N-1)/2$ distances.
We have, moreover, evaluated the network *efficiency E*, defined as in [12]

$$E = 1 / N(N-1) \sum_{ij} 1/d_{ij} \tag{2}$$

where d_{ij} is the distance between nodes *i* and *j*. The quantity *E* generalizes the concept of "small world" [4,5] for any network, regardless of its topology.

We have also evaluated some property deduced by the spectral analysis. There is a wide literature on the spectral analysis of the **A** and **L** matrices associated to a graph [10,13,14]. As in a previous attempt, made to analyze the network of the Internet AS-level routers [15], we focus on a specific result derived by the spectral analysis of the **L** matrix and the so-called "min cut" theorem [16-18]. This can be stated as follows. The lowest eigenvalue λ_1 of the **L** matrix is always vanishing ($\lambda_1=0$) and the orthonormalized components of its associated eigenvector $\mathbf{v}^L{}_\mathbf{1}$ are all equal to $1/\sqrt{N}$ (*N* being the number of nodes). The components of the second eigenvector $\mathbf{v}^L{}_\mathbf{2}$ associated to the second eigenvalue λ_2 of the Laplacian ($\lambda_2\neq0$) have, in turn, different signs. The "min-cut" theorem provides a recipe which, starting from the analysis of the sign of the components of the second eigenvector $\mathbf{v}^L{}_\mathbf{2}$ of **L,** allows the partition of the network into two nearly equal sub-networks connected via the minimum number of connections. The first sub-network is formed by nodes with a positive component, the second by those with a negative component. The "min cut" theorem ensures that the number of links to be cut n_l, between these two set is the smallest possible cut between two nearly equal subset of the whole graph, i.e. the cut has a minimum "weight". We have thus defined n_l as the number of links joining nodes belonging to the different sub-networks (i.e. from the total number of links *L*, we count only those joining nodes belonging to different sub-networks). The links defined by the "min cut" algorithm are those whose failure would induce the "most effective" perturbation to the network by producing the maximum number of "effective broken links". The values of n_l , resulting from the spectral analysis of the network's Laplacians are reported in the last column of Table 1.

network	*N*	*L*	□□□	*c*	k_{max}	*d*	$<l>$	*E*	n_l
G_1 (I-380 kV)	127	171	2.69	0.156	7	25	8.47	0.173	3
G_2 (F-400 kV)	146	223	3.05	0.279	8	15	6.61	0.197	7
G_3 (S-400 kV)	98	175	3.57	0.316	9	11	4.92	0.259	7
G_4(I-fine grain)	1926	2240	2.33	0.019	15	42	14.78	0.082	9

Table 1. General properties of the analysed networks. *N* is the number of nodes, *L* the number of links, $<k>$ the average degree; *c* the average clustering coefficient; k_{max} the degree of the node with the maximum number of connections (hub); *d* is the network diameter, $<l>$ the characteristic path length; *E* is the network's *efficiency* [12]; n_l the number of links connecting the two sub-networks solution of the "min-cut" problem.

Data reported in Tab.1 provides a picture of the coarse-grain electric transmission lines as sparse networks (low ratio *2L/[N (N-1)]*) with a low average degree ($<k> \leq 3.6$), a quite low value of the maximum degree (which ranges between 7 and 9) but with a quite large clustering ($0.156 < c < 0.310$). The networks, moreover, display characteristic path lengths (between 5 and 8.5) larger than those corresponding to random networks ($<l_{rand}>=logN/log<k>$), with maximal values as large as 25 (in the I-380 kV case).

A specific interest for the network's classification is assumed by the distribution and the cumulative distributions of the specific topological quantities. We have firstly evaluated the degree's distribution and its cumulative distribution to compare data from the considered networks with those coming from the inspection of transmission lines in other countries recently reported [3]. This work, dealing with the US electrical power networks, has pointed out the specific exponential shape of the cumulative distribution of node's degree electrical power transmission lines. The cumulative distribution P(k>K) is defined as the probability of having a node whose degree is higher than a given value of K. Our estimates for the four networks, P_1,P_2, P_3 and P_4 (displayed in fig.1) have allowed to find

$$P_1(k>K) = \exp(-0.18\ K^2) \quad (3)$$
$$P_2(k>K) = \exp(-0.21\ K^2) + 0.18 \exp[-0.25\ (K-4.0)^2] \quad (4)$$
$$P_3(k>K) = 0.96 \exp(-0.17\ K^2) + 0.25 \exp[-0.19\ (K-3.9)^2] \quad (5)$$
$$P_4(k>K) = \exp(-0.32\ K^2) + 0.065 \exp[-0.20\ (K-4.0)^2] \quad (6)$$

which are different from those reported in [3]. There are several points, however, which lead the european networks to be somehow different from the US one. The latter, in fact, is huge containing more than 10^4 nodes, with degree values ranging up to 25. The considered networks are much smaller, with the maximum degree value smaller than 10. The low value of the highest degree is such to produce a gaussian-type decay of the cumulative distribution of the degree.

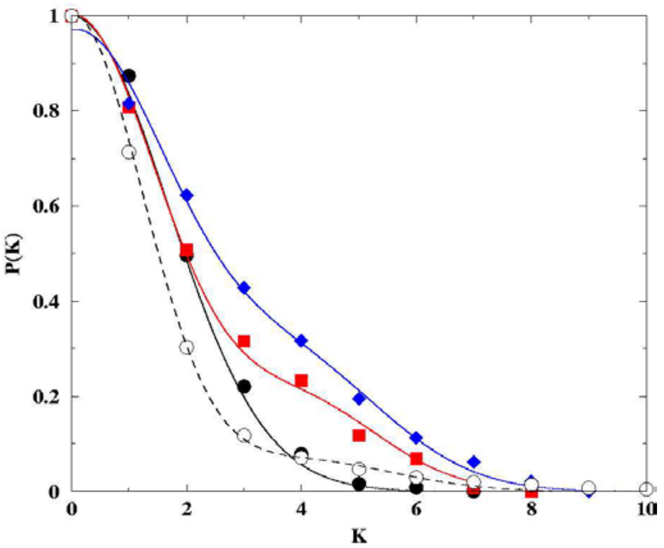

Fig 1. Cumulative distribution of the node's degree of the G_1 (full circles), G_2 (squares) and G_3 (diamonds) networks: lines correspond to the best fits (see, e.g., eqs.4 -6). The G_4 network (empty circles) has a best fit represented by eq.7.

We have evaluated the distribution of the lengths of the shortest paths, whose shape is reported in fig.2.

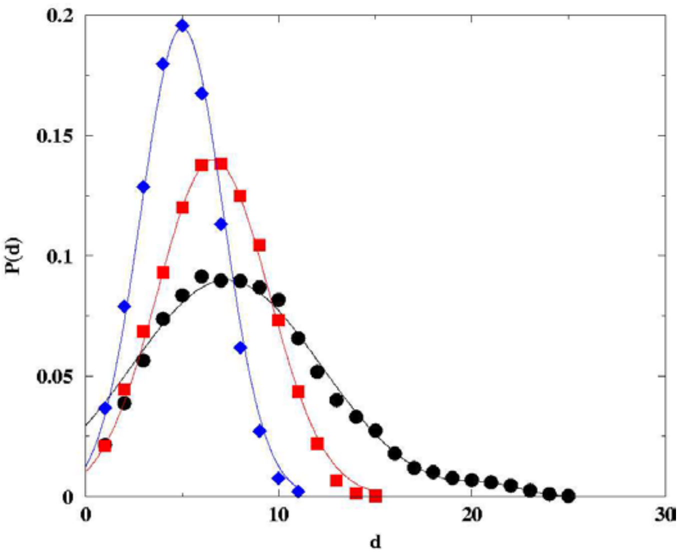

Fig 2. Distribution of the shortest path lengths between all the nodes of the G_1 (dots), G_2 (squares) and G_3 (diamonds) networks: lines correspond to the best fits obtained by using the following gaussian functions:
$P_1(d)=0.09e^{-0.021(d-7.3)^2}+ 0.004e^{-0.2(d-21)^2}$, $P_2(d)=0.14\ e^{-0.06(d-6.61)^2}$, $P_3(d)=0.195\ e^{-0.11(d-5)^2}$

It is interesting to notice how the peculiar geography of the country is also reflected into these data. In fig.2, for instance, the distribution of the node's distance in G_1=I-380 kV is bi-modal, with the two best-fit gaussians being centered at d_1=7.3 and d_2=21. This is still related to the form of the italian country; the low d_1 value averages the distances between nodes belonging to the same geographical district, the large d_2 value, in turn, is the average distance between nodes belonging to different districts. The scarce number of shortcuts between North and South compels the use of the few longitudinal links and that of a serial connection, thus resulting in large internode distances.

We have attempted to give a major emphasis to the issues of network's vulnerability. We have associated the concept of network vulnerability to a number of different quantities: the extent of "critical sections" present in the network and the conditional probability of disconnecting nodes given the removal of a number of links [19].

We have defined "critical sections" the set of links associated to a network partition made on the basis of the min-cut procedure. As it has been previously explained, the min-cut procedure is able to divide the network into two sub-networks whose interconnection has the minimum weight (i.e. is composed by the minimum number of links n_l,). Lower the value of n_l, higher the vulnerability of the network (when the links of a given critical section are removed, the two sub-networks resulting from the cut are totally disconnected). If we apply the procedure to the Italian G_1 network, it results to be divided in two nearly equal sub-networks, the first containing 51 nodes, the second 76 nodes. The number of links connecting these two halves is n_l=3; the position of the related links (whose set will be referred to as "first-level critical section") in the network is reported in Fig.3 (thick black links). If we take the resulting two sub-networks and we repeat the min-cut procedure on each of them, we will identify two "second-level critical sections"; the relative links are reported in Fig.3 in thick blue lines. It is interesting to notice that both first- and second-level critical sections have comparable extent (the values of n_l for the second-level critical sections are equal to 3 and 4, see fig.3). This result reflects a multi-scale vulnerability of the considered network. In fact, the vulnerability of the network (low n_l value) extends to different sections of different sizes.

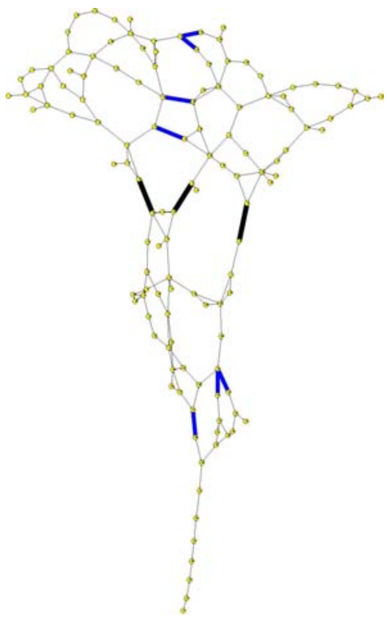

Fig 3. G_1 network after two successive applications of the min-cut procedure: thick black links refer to the first critical section resulting from the first iteration; thick blue links refer to the critical sections resulting from the successive application of the min-cut procedure to the two sub-networks defined by the first application of the min-cut.

Low values of n_l of first-level critical sections have been also reported for early stages of growth of the US AS-level internet routers [15]; however, later stages of growth of that network have been found to be characterized by a much larger value of n_l, thus indicating a very rapid increase of n_l (its growth has been estimated to be 20 times faster than the corresponding growth of the number of nodes). The growth mechanism which has been proven to be consistent with the resulting figures of the principal network's properties is the so-called "Triad formation" mechanism [20]. Large n_l values are thus an emerging feature of unconstrained network growths; this property seems to be associated to some fundamental property needed to ensure a correct functioning to the network. An immediate short-cut could be envisaged between the n_l value and the network vulnerability. The following "computer experiment" has been realized to assess the existence of a true functional relation between n_l and that property.

The G_1, G_2 and G_3 networks have been submitted to a controlled links removal, to determine the extent of the *damage* introduced by this action. The *damage* resulting from the cut of 1,2,..*r* links has been defined as the number of nodes which result to be disconnected upon such action. This results is evaluating the conditional probability $P(k|r)$ that *k* nodes become unreachable upon the cut of *r* links of the network. This test has been carried out by suppressing *r* links and by subsequently recalculating the distance among all the different *i-j* couples of the networks. If a node cannot be reached by any of the others, that node is defined as "disconnected" and the value which quantifies the extent of the introduced damage is augmented by one. The tests with $r = 1$ have been carried out exhaustively: one at a time, each link has been suppressed and the number of disconnected nodes has been evaluated after each removal. The link has been then reintroduced, a further link suppressed and the procedure iterated. For $r \geq 2$, a sample of $2 \cdot 10^3$ *r*-ples of links have been randomly selected; the reported results refer to an average over these samples. Fig.4, fig.5 and fig.6 report the conditional probability $P(k|r)$ in the three networks. This is a robustness index allowing the measure of the extent of the network resistance to faults such as the loss of an *r*-uple of links. This quantity is a clear indication of the level of vulnerability of the network versus the action of links removal. In the analyzed cases, results point to a substantial vulnerability of the G_1 network (fig.5) as, even in the case of the removal of r=2 links, there is a considerable probability P(k>2|2)~5.8% of having k>2 disconnected nodes (in the G_2 case, P(k>2|2) ~0.3%). In turn, the G_2 and G_3 networks show a quite higher robustness as the probability P(k>2|7) ≤ 3 % (to have k>2 disconnected nodes upon removal of r=7 links, see figs.5 and 6).

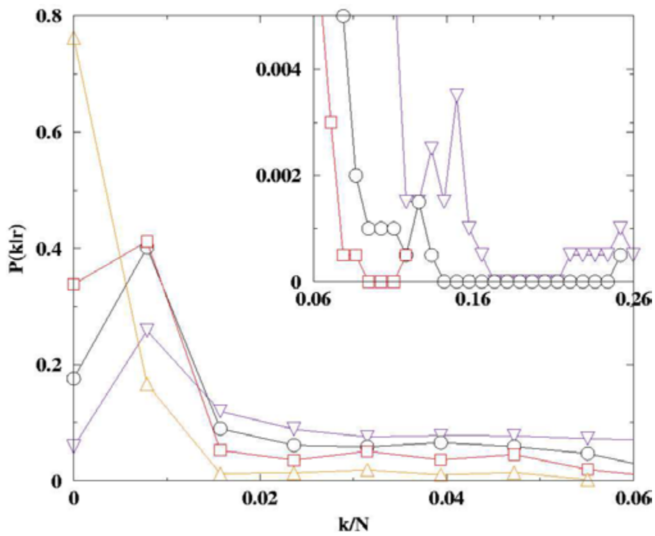

Fig 4. Conditional probability P (k|r) of having k nodes disconnected given r links removed, in the G_1 network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa is normalized over the number of nodes. Legenda of symbols: triangle up r=2; square r=7; circle r=10; triangle down r=15.

Results in figs.4-6 are reported in terms of the fraction of nodes disconnected upon the removal of r links. The normalization of this datum allows to better compare the different impact of the fault event on the networks. Also this property seems to be related to the specific geography of the country. In fact, as the network design has to be tightly mapped onto the country morphology and its peculiar distribution of sources and loads, the resulting structure of the network unavoidably exhibits some weakness. In the case of the italian network, the specific morphology, with a narrow peninsular body, ""squeezes" the structure of the network. In the region where the continental, north part of the country becomes a peninsula and the east-west extension of the country reduces to a few hundreds kilometers, the network shows a weakness related to the lack of a robust interconnection between the north and the central-south parts. In fact, the min-cut analysis of the G_1 network shows that the narrowing of the country (and, consequently, of the network) reduces of the number of links; this fact, in some sense, eases the cut of the networks into two almost equal sections. This division moreover establishes by only three links, thus becoming a severe source of weakness in the network. This effect is almost absent in France and Spain where the countries morphology prevents its occurrence (n_l =7 in both cases).

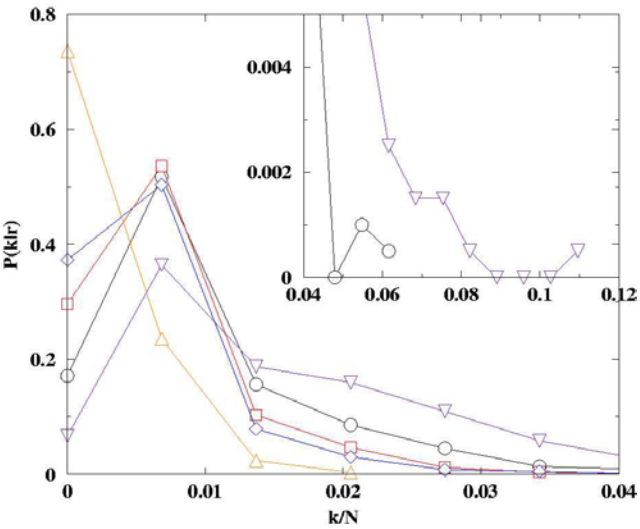

Fig 5. Conditional probability P (k|r) of having k nodes disconnected given r links removed, in the G_2 network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa is normalized over the number of nodes. Legenda of symbols: triangle up r=2; diamond r=6; square r=7; circle r=10; triangle down r=15.

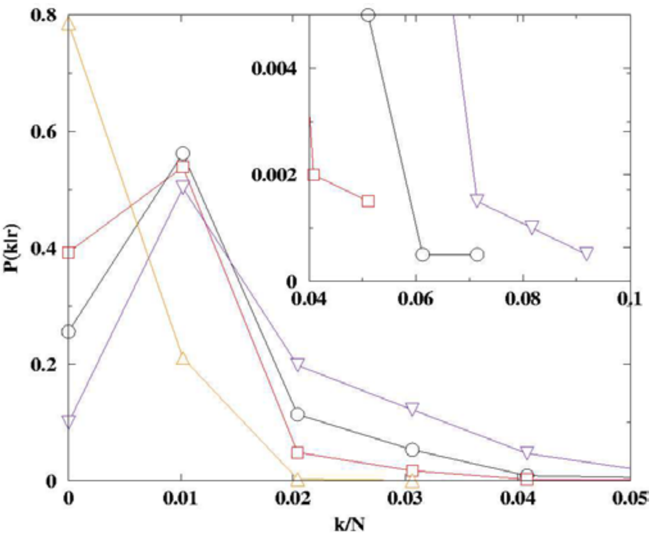

Fig 6. Conditional probability P (k|r) of having k nodes disconnected given r links removed, in the G_3 network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa are normalized over the number of nodes. Legenda of symbols: triangle up r=2; square r=7; circle r=10; triangle down r=15.

To show a possible relation between vulnerability and the value of n_l , we have attempted to "optimize" the structure of the G_1 network to increase the value of n_l resulting from the min-cut procedure. This has been attempted by inserting just one extra link in the network and by recording the resulting variation of n_l. and that of the vulnerability, expressed in terms of the variation of the P(k|r) function. The optimization procedure ended up with the following solution: if two specific nodes are connected (one in the centre-north, the other in the centre-south), the number of links solution of the min-cut procedure n_l rises from n_l =3 to n_l =12.

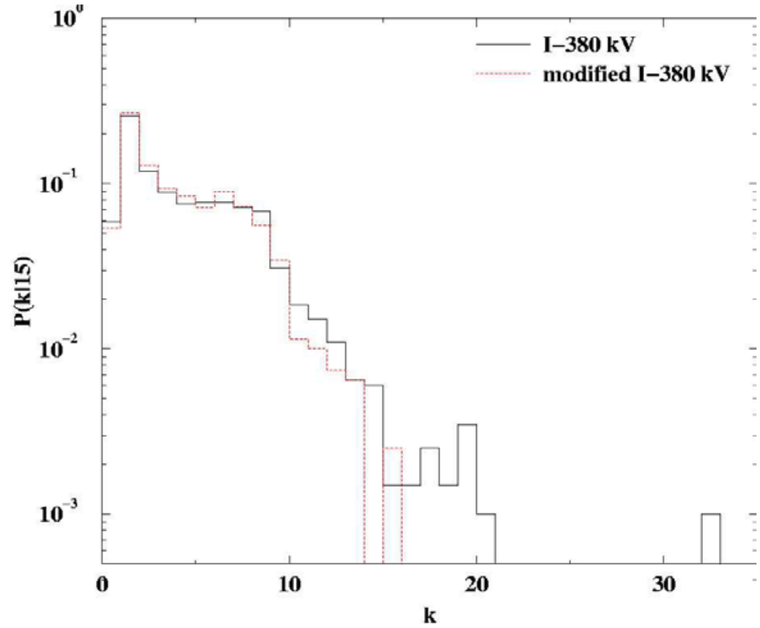

Fig 7. Linear-log plot of the conditional probability P(k|15) of having k nodes disconnected given $r=15$ links removed, in the G_1 network: full line is the original I-380 kV network, dotted line is the same network with a new extra link (which brings to a maximum the number of links solution of the min-cut problem) added.

We have also shown that this specific action is also able to decrease the network vulnerability. In fact, if we evaluate the conditional probability P(k|r) in the modified network, we show how the simple introduction of that new link, is able to substantially reducing the vulnerability of the network: in fact, as r=15, there is a low probability to have a number of disconnected links k>15 in the modified network, while in the unperturbed G_1 network, there is a non-vanishing probability of disconnecting up to 33 links (fig.7). The same holds for r<15 even if the effect is less pronounced.

In fig.8 we present the graphs of the G_1 network as modified by the introduction of the further link able to bring the value of n_l =12.

3. Conclusions

This work has highlighted the role that topological analysis might play in assessing several properties of a complex network, particularly those related to its robustness and fault-tolerance. These methods, when combined to spectral analysis, allow to producing an almost complete description of the *static response* of the network (for *static response* we intend the properties of the network which are a direct consequence of its specific structure of nodes and links). A number of other properties might be disclosed by a *dynamic* analysis of the networks, where the adjacency matrix is substituted by a load (or capacity) matrix. This can be used to evaluate dynamical series of events which may be related to some overload of the network's components induced by some faults. This analysis, which will adapt, to the specific problem, a theoretical approach recently proposed [21], will be the object of a further investigation.

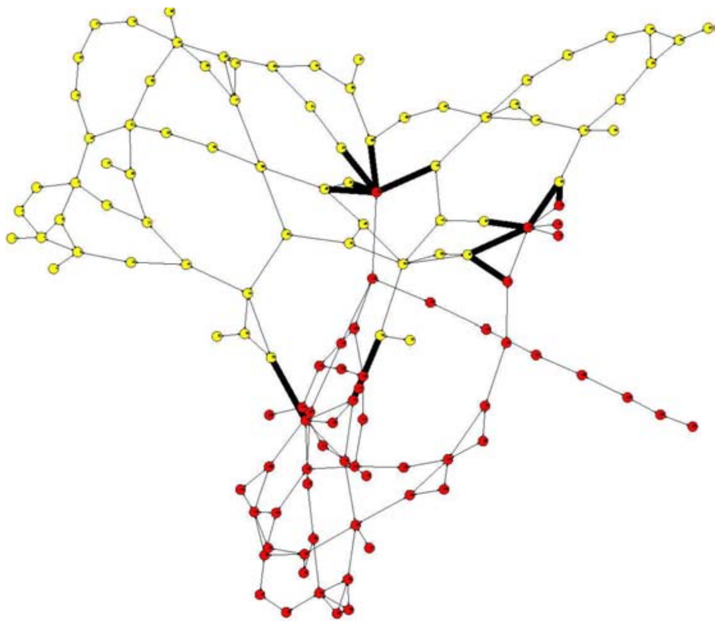

Fig 8. Graphs of the G_1 network after the placement of the extra link which brings the min-cut number n_l from n_l =3 to n_l =12 . Links defined by the new min-cut solution are highlighted. Nodes of different colors belong to the different sub-networks.

This analysis, performed on three european high-voltage transmission networks, has located a number of properties which are common to these networks, independently by their structure, and others which are rather peculiar, somehow reflecting the specific geography of the country.

Data shows that the coarse-grain networks display topological properties which are typical of "small world" systems. In fact they exhibit a very large clustering coefficient (much higher than the corresponding value in random networks $c_{rand} = <k>/N$) and a large characteristic path length, larger than that pertaining to random networks. This fact has been already noticed by other authors who have analyzed the US electrical transmission lines [4]. Data of the G_4 network (seen as the G_1 at a later stage of growth) confirm the hypothesis that "constrained- growth" mechanisms are not able to improve network's properties: clustering decreases and the value of n_l has only a small increase. These growth's data should be compared to those resulting from an unconstrained growth, such as that experienced by the US AS-level router [15].

The vulnerability analysis performed on the basis of two indicators (the size of the links comprised in the critical sections and the conditional probability of having nodes disconnected upon links removal) has identified several weaknesses in the considered networks.

In conclusion, it must be recall, however, that the analysis of the network's topology is able to cope only with a single part of the problem. In fact, sequence-of-events analyses of the largest blackouts experienced in 2003 have shown that, although being triggered by events occurred on the *physical* network, most of the resulting effects have been a consequence of errors and faults at the *organizational* and the *cyber-control* levels [22], made when attempting to heal the effects produced by the initial events. This confirms the validity of a global strategy aimed at producing reliable tools for efficiently supporting human and/or cyber decisions when dealing with unpredictable faults, with the further constraint of being rapid and maximally effective. The combination of the predictions, made on the basis of a topological analysis of the network, of its static and dynamic response to perturbations, might provide a first-level insight to be used into a new generation of *what-if* toolsets for decision-support at the *cyber-control* level. The realization and the use of tools for *what-if* analysis has been strongly encouraged by both the US/Canada Task force [1] and by european UCTE [2, 23] for preventing (or better governing) power outages such as those, occurred in US and in Italy, which might produce unpredictable social and economic drawbacks.

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